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Generalized Entropy for Intuitionistic Fuzzy Sets

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ABSTRACT

Information theory was introduced by Shannon in 1948 and it includes the study of uncertainty measures. Analogous to information theory which is founded on probability theory, notion of fuzzy sets was established by Zadeh in 1965 which is a mechanism to manage uncertainty. The pedestal for present study is information theory with intuitionistic fuzzy sets. Atanassov (1986) devised Intuitionistic fuzzy sets which is an expedient devise to deal vagueness and uncertainty. In present study, we extended the fuzzy entropy proved by Gupta et al.(2014) and then generalized it for intuitionistic fuzzy sets with axiomatic justification and presented importance of parameter α . Further, some properties of this measure are analyzed and the performance of proposed entropy measure at different values of α on the basis of linguistic variables is compared with the help of a numerical example.

Keywords: Fuzzy entropy, intuitionistic fuzzy entropy, exponential intuitionistic fuzzy entropy, linguistic variables.

1. Introduction

Shannon (1948) defined measure of uncertainty for discrete and continuous probability distribution and proved various mathematical properties corresponding to both discrete and continuous probability distributions. Parallel to probability theory Zadeh discovered Fuzzy set theory in 1965. Zadeh (1965) introduced a measure of fuzzy information termed as fuzzy entropy which is based on Shannon entropy. In fuzzy set theory, non-membership value of a member is complement of its membership value from one, but practically it is not true, this is dealt by higher order Fuzzy set proposed by Atanassov (1986) and is termed as intuitionistic fuzzy sets (IFSs). IFSs are advantageous in handling imprecision as well as hesitancy originated from inadequate information. It characterizes two characteristic functions for membership, $\mu_A(x)$ and non-membership, $\nu_A(x)$ for an element x belonging to the universe of discourse. Later, Burillo and Bustince (1996) defined the distance measure among IFSs. Also gave axiomatic definition of intuitionistic fuzzy entropy with its characterization. Thereafter, Szmidt and Kacprzyk (2001, 2005 a, b) extended the fuzzy entropy properties proposed by De Luca and Termini (1972) and abridged the distance calculated using Hamming distance. Hung and Yang (2006), Vlachos and Sergiadis (2007), Chaira and Ray (2008), Ye (2010) and Verma and Sharma (2013) developed entropies for IFS and discrimination measure between IFSs with corresponding properties and demonstrated the efficiency in the framework of medical diagnosis, pattern recognition, edge detection, image segmentation and multi-criteria decision making. The article initially consist a brief introduction about IFSs followed by α exponential intuitionistic fuzzy entropy corresponding to α exponential fuzzy entropy. Further, to check the performance of proposed intuitionistic entropy measure on the basis of linguistic variables a numerical example is used.

2. Brief Introduction about Intuitionistic Fuzzy Sets.

Atanassov (1986) proposed a generalization of fuzzy sets characterized as Intuitionistic Fuzzy Sets (IFSs). It is has dealt with vagueness and hesitancy originated from inadequate information with expediency. It characterizes two characteristic functions for membership and non-membership $\mu_A(x)$ and $\nu_A(x)$ respectively for an element x belonging to the universe of discourse. An IFS is given as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in \Omega\}$ where $\mu_A : \Omega \to [0,1]$ and $\nu_A : \Omega \to [0,1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $\mu_A(x)$ and $\nu_A(x) \in \Omega$ denote degree of membership and non-membership of $x \in A$, respectively. For each IFS in Ω,

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ denotes index of hesitancy or intuitionistic fuzzy index. Obviously, $0 \leq \pi_A(x) \leq 1$.The properties of fuzzy entropy proposed by De Luca and Termini(1972) were extended by Szmidt and Kacprzyk (2001) for an entropy measure in IFSs are as follows:

1. Intuitionistic fuzzy entropy is zero iff set is crisp.
2. Intuitionistic fuzzy entropy is one iff membership value is same as non-membership values for every element.
3. Intuitionistic fuzzy entropy decreases as set get sharpened i.e if A is less fuzzy than B it implies that $E(A) \leq E(B)$.
4. Intuitionistic fuzzy entropy of a set is same as its complement.

Li, Lu and Cai (2003) suggested a method of converting intuitionistic fuzzy sets to fuzzy sets by allocating hesitation degree equally to membership and non-membership values. In subsequent section, a generalized entropy measure for intuitionistic fuzzy sets have been proposed which is based on α exponential fuzzy entropy given by Gupta et al. (2014) as represented by equation (1) and validated some axioms for the same.

$$\begin{aligned} H_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}} - 1)} \sum_{i=1}^{n} [\mu_A^\alpha(x_i)e^{1-\mu_A^\alpha(x_i)} \\ + (1 - \mu_A(x_i))^\alpha e^{1-(1-\mu_A(x_i))^\alpha} - 1], \\ 0 < \alpha \leq 1 \end{aligned} \tag{1}$$

3. Extension of α Exponential Intuitionistic Fuzzy Entropy

In this section, we have extended the result given by Gupta et al. (2014) as represented by equation (1). Let

$H_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1))} \sum_{i=1}^{n} H_\alpha^i(A)$
where $H_\alpha^i(A) = [\mu_A^\alpha(x_i)e^{1-\mu_A^\alpha(x_i)} + (1 - \mu_A(x_i))^\alpha e^{1-(1-\mu_A(x_i))^\alpha} - 1]$

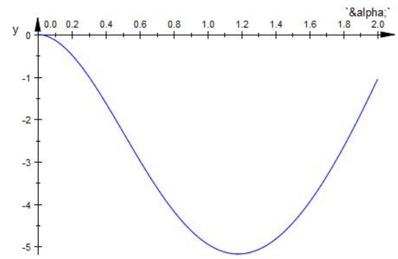

Figure 1: $\frac{\partial^2 H_\alpha^i(A)}{(\mu_A(x_i))^2} at\ \mu_A(x_i)) = \frac{1}{2}$

$$\begin{aligned}\frac{\partial^2 H_\alpha^i(A)}{\partial(\mu_A(x_i))^2} &= [\alpha(\alpha-1)\mu_A^{\alpha-1}(x_i)e^{1-\mu_A^\alpha(x_i)} + \alpha^2(1-3\alpha)\mu_A^{2\alpha-2}(x_i)e^{1-\mu_A^\alpha(x_i)} \\ &+ \alpha(1-3\alpha)(1-\mu_A(x_i))^{2\alpha-2}e^{1-(1-\mu_A(x_i))^\alpha} \\ &+ \alpha^2 e^{1-(1-\mu_A(x_i))^\alpha}(1-\mu_A(x_i))^{3\alpha-2} \\ &+ \alpha(\alpha-1)e^{1-(1-\mu_A(x_i))^\alpha}(1-\mu_A(x_i))^{\alpha-2} \\ &+ \alpha^2\mu_A(x_i)^{3\alpha-2}e^{1-\mu_A(x_i)^\alpha}]\end{aligned} \tag{2}$$

At $\mu_A(x_i)) = \frac{1}{2}$ equation (2) reduces to

$$\begin{aligned}\frac{\partial^2 H_\alpha^i(A)}{\partial(\mu_A(x_i))^2} &= \alpha e^{1-0.5^\alpha}(21.74625463\alpha 0.5^{3\alpha} \\ &+ 21.74625463\alpha 0.5^\alpha + 21.74625463\alpha 0.5^{2\alpha} \\ &- 21.74625463\alpha 0.5^\alpha - 65.23876388\alpha 0.5^{2\alpha})\end{aligned}$$

And $\frac{\partial^2 H_\alpha^i(A)}{\partial(\mu_A(x_i))^2} < 0$ at $\mu_A(x_i)) = \frac{1}{2}$ when $0 < \alpha \leq 2$ as shown in Figure (1) Hence equation (1) holds for $0 < \alpha \leq 2$.

4. α Exponential Intuitionistic Fuzzy Entropy

We define α exponential intuitionistic fuzzy entropy corresponding to (1), using method proposed by Li, Lu and Cai (2003). According to this method, IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in \Omega\}$ is transformed in to fuzzy set (A^*) having membership function $\mu_{A^*}(x) = \mu_A(x) + (\pi_A(x))/2 = \frac{\mu_A(x)+1-\nu_A(x)}{2}$.Thus parallel to α exponential fuzzy entropy, we introduce α exponential intuitionistic fuzzy entropy defined as follows:

$$K_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha} + (1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha} - 1]$$

for $0 < \alpha \leq 2$.

$$K_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(\frac{1-\mu_A(x_i)+\nu_A(x_i)}{2})^\alpha} + (1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha} - 1] \quad (3)$$

for $0 < \alpha \leq 2$. In order to prove that (3) is a valid entropy measure for intuitionistic fuzzy sets, four properties given by Szmidt and Kacprzyk (2001) has been checked as follows:

1. Property 1: Consider a crisp set A i.e. membership values of elements are either 0 or 1. Then $K_\alpha(A) = 0$. By putting $\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} = \gamma_A(x_i)$, equation (3) reduces to

$$K_\alpha(A) = \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[\gamma_A^\alpha(x_i)e^{1-\gamma_A^\alpha(x_i)} + (1-\gamma_A(x_i))^\alpha e^{1-(1-\gamma_A(x_i))^\alpha} - 1]$$

   for $0 < \alpha \leq 2$

   which is same as equation (1). Therefore by the properties of fuzzy entropy, fuzzy entropy becomes zero if and only if $\gamma_A(x_i) = 0$ or 1 for all i.
   $\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} = 0$

$$\Rightarrow \mu_A(x_i) - \nu_A(x_i) = 1 \quad (4)$$

   for all i.Again
   $\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2} = 0$

$$\Rightarrow \nu_A(x_i) - \mu_A(x_i) = 1 \quad (5)$$

   for all i. Also,

$$\mu_A(x_i) + \nu_A(x_i) \leq 1 \quad (6)$$

From equation (4) and (6), we get that $\mu_A(x_i) = 0, \nu_A(x_i) = 1$. From equation (5) and (6), we get that $\mu_A(x_i) = 1, \nu_A(x_i) = 0$. Conversely, if $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\mu_A(x_i) = 1, \nu_A(x_i) = 0, K_\alpha(A)$ clearly reduces to zero. Hence $K_\alpha(A) = 0$ iff either $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\mu_A(x_i) = 1, \nu_A(x_i) = 0$ for all i.

2. Property 2: Let $\mu_A(x_i) = \nu_A(x_i)$ for all i. Then equation (3) reduces to 1 i.e. $K_\alpha(A) = 1$.Let $K_\alpha(A) = \frac{1}{n}\sum_{i=1}^{n} F(\gamma_A(x_i))$, $0 < \alpha \leq 1$.

   Where$F(\gamma_A(x_i)) = \frac{[(\gamma_A(x_i))^\alpha e^{1-(\gamma_A(x_i))^\alpha} + (1-\gamma_A(x_i))^\alpha e^{1-(1-\gamma_A(x_i))^\alpha} - 1]}{n(2^{1-\alpha}e^{1-2^{-\alpha}} - 1)}$

   Thus, if $K_\alpha(A) = 1, \Rightarrow \frac{1}{n}\sum_{i=1}^{n} F(\gamma_A(x_i)) = 1$

$$\Rightarrow F(\gamma_A(x_i)) = 1 \forall i. \tag{7}$$

   Differentiating equation (7) w.r.t $\gamma_A(x_i)$

$$\frac{\partial F(\gamma_A(x_i))}{\partial \gamma_A(x_i)} = \frac{\begin{array}{l}[(\alpha\gamma_A(x_i))^{\alpha-1} e^{1-(\gamma_A(x_i))^\alpha}(1-(\gamma_A(x_i))^\alpha) \\ + \alpha(1-\gamma_A(x_i))^{\alpha-1} e^{1-(1-\gamma_A(x_i))^\alpha}(1-\gamma_A(x_i))^\alpha]\end{array}}{n(2^{1-\alpha}e^{1-2^{-\alpha}} - 1)} \tag{8}$$

   $\frac{\partial F(\gamma_A(x_i))}{\partial \gamma_A(x_i)} = 0$, so

$$\begin{aligned}\Rightarrow &[(\alpha\gamma_A(x_i))^{\alpha-1} e^{1-(\gamma_A(x_i))^\alpha}(1-(\gamma_A(x_i))^\alpha) \\ &+ \alpha(1-\gamma_A(x_i))^{\alpha-1} e^{1-(1-\gamma_A(x_i))^\alpha}(1-\gamma_A(x_i))^\alpha] = 0\end{aligned} \tag{9}$$

   Equation (9) holds when$\gamma_A(x_i) = \frac{1}{2}$ Again differentiating equation (8), we get

$$\frac{\partial^2 F(\gamma_A(x_i))}{\partial(\gamma_A(x_i))^2} = \frac{\begin{array}{l}[(\alpha\gamma_A(x_i))^{\alpha-1} e^{1-(\gamma_A(x_i))^\alpha}(\alpha-1) \\ + \alpha^2(\gamma_A(x_i))^{2\alpha-2} e^{1-(\gamma_A(x_i))^\alpha}(1-3\alpha) \\ + \alpha^2(1-\gamma_A(x_i))^{3\alpha-2} e^{1-(1-\gamma_A(x_i))^\alpha} \\ + \alpha(1-3\alpha)(1-\gamma_A(x_i))^{(2\alpha-2)} e^{1-(1-\gamma_A(x_i))^\alpha} \\ + \alpha(\alpha-1)(1-\gamma_A(x_i))^{\alpha-2} e^{1-(1-\gamma_A(x_i))^\alpha} \\ + \alpha^2(\gamma_A(x_i))^{3\alpha-2} e^{1-(\gamma_A(x_i))^\alpha}]\end{array}}{n(2^{1-\alpha}e^{1-2^{-\alpha}} - 1)}$$

   For all i and $0 < \alpha \leq 2$. $\frac{\partial^2 F(\gamma_A(x_i))}{\partial(\gamma_A(x_i))^2} \leq 0$ at$\gamma_A(x_i) = \frac{1}{2}$ as represented in figure 2 (where $y = \frac{\partial^2 F(\gamma_A(x_i))}{\partial(\gamma_A(x_i))^2}$ and $alpha = \alpha$) Hence $F(\gamma_A(x_i))$ is a concave function obtaining its maximum value at $\gamma_A(x_i) = \frac{1}{2}$ i.e. $\gamma_A(x_i) = \frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}$. Thus $K_\alpha(A)$ attains maxima at $\mu_A(x_i) = \nu_A(x_i)$ for all i.

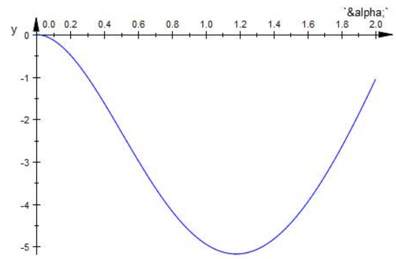

Figure 2: $\frac{\partial^2 F(\gamma_A(x_i))}{\partial(\gamma_A(x_i))^2}$

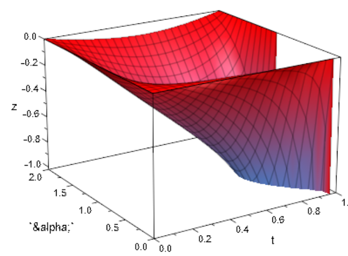

Figure 3: $\frac{df}{dt}$

3. Property 3: In order to prove that property 3 holds. It is enough to show that the function $f(a,b) = [(\frac{a+1-b}{2})^\alpha e^{1-(\frac{(a+1-b)}{2})^\alpha} + (\frac{1-a+b}{2})^\alpha e^{1-(\frac{(1-a+b)}{2})^\alpha} - 1]$ Where $a, b \in [0,1]$, is increasing w.r.t a and decreasing for b.In order to determine critical point of f, equate $\frac{\partial f}{\partial a} = 0$ and $\frac{\partial f}{\partial b} = 0$ we get a=b. Again, let$f(a,b) = [(\frac{a+1-b}{2})^\alpha e^{1-(\frac{(a+1-b)}{2})^\alpha} + (\frac{1-a+b}{2})^\alpha e^{1-(\frac{(1-a+b)}{2})^\alpha} - 1]$, where $a > b$, Put $t = a - b > 0, 0 < \alpha \leq 2$ then
$f(t) = [(\frac{t+1}{2})^\alpha e^{1-(\frac{(1+t)}{2})^\alpha} + (\frac{1-t}{2})^\alpha e^{1-(\frac{(1-t)}{2})^\alpha} - 1]$
$\frac{\partial f}{\partial t} = [\frac{\alpha(1+t)^{\alpha-1}(1-(1+t)^\alpha)e^{-\frac{(1+t)^\alpha - 2^\alpha}{2^\alpha}}}{2^\alpha} - \frac{\alpha(1-t)^{\alpha-1}(1-(1-t)^\alpha)e^{2^\alpha - \frac{(1-t)^\alpha}{2^\alpha}}}{2^\alpha}] < 0$
as shown in figure 3. and thus function f (t) is a decreasing when $t = a - b > 0$. Similarly, it can proved that when$a < b, t = b - a > 0$ then f(t) is decreasing function. Hence$\frac{\partial f}{\partial a} \leq 0$, when $a > b$ and $\frac{\partial f}{\partial a} \geq 0$, when $a < b$ .Similarly, $\frac{\partial f}{\partial b} \geq 0$,when $a > b$ and $\frac{\partial f}{\partial b} \leq 0$, when $a < b$.Thus by monotonicity of function f(t) , property (2) and containment property of IFSs, we get that $K_\alpha(A) \leq K_\alpha(B)$ when,$A \subseteq B$.

4. Property 4 :Clearly it holds by description of complement of intuitionistic fuzzy sets and equation (2).i.e. $K_\alpha(A) = K_\alpha(\bar{A})$.

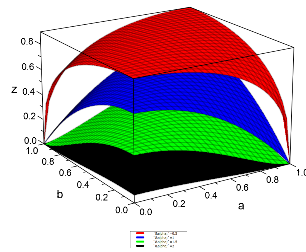

Figure 4: Comparison of Intuitionistic Fuzzy Entropy at different values ofα

Hence $K_\alpha(A)$ is a valid entropy measure for intuitionistic fuzzy sets.
It is observed that kurtosis of $K_\alpha(A)$ varies as the value of generalization parameter α assumes different values within the specified range as shown in figure 4. Thus addition of a parameter makes distribution more flexible and affluent for practical purposes. It has also been supported by Andargie and Rao(2013) so the suitable value of generalization parameter α assists practical modeling of data.

The proposed measure has following properties:

Theorem 4.1. *Consider two intuitionistic fuzzy sets A and B then*
$K_\alpha(A \cup B) + K_\alpha(A \cap B) = K_\alpha(A) + K_\alpha(B)$
$holds for K_\alpha(A).$

Proof. Universe of discourse can be divided into two subsets as $\Omega = \{x \in X \mid \mu_A(x) \leq \mu_B(x)\} \cup \{x \in X \mid \mu_A(x) \geq \mu_B(x)\}$, which we will denote as Ω_1, Ω_2 respectively. In $\Omega_1, \mu_A(x) \leq \mu_B(x) \Rightarrow \nu_A(x) \geq \nu_B(x)$, then
$\mu_{(A\cup B)}(x) = max\{\mu_A(x), \mu_B(x)\} = \mu_B(x),$
$\nu_{(A\cup B)}(x) = min\{\nu_A(x), \nu_B(x)\} = \nu_B(x)$
$\mu_{(A\cap B)}(x) = min\{\mu_A(x), \mu_B(x)\} = \mu_A(x),$
$\nu_{(A\cap B)}(x) = max\{\nu_A(x), \nu_B(x)\} = \nu_A(x).$

So $A \cup B = \{\langle x, \mu_{(A\cup B)}(x), \nu_{(A\cap B)}(x)\rangle, x \in \Omega_1\}$. From equation (2), we have

$$K_\alpha(A \cup B)$$
$$= \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}} - 1)} \sum_{i=1}^{n} [(\frac{\mu_{A\cup B}(x_i) + 1 - \nu_{A\cup B}(x_i)}{2})^\alpha e^{1-(\frac{1-\mu_{A\cup B}(x_i)+\nu_{A\cup B}(x_i)}{2})^\alpha}$$
$$+ (1 - \frac{\mu_{A\cup B}(x_i) + 1 - \nu_{A\cup B}(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_{A\cup B}(x_i)+1-\nu_{A\cup B}(x_i)}{2})^\alpha} - 1]$$

For $0 < \alpha \leq 2$.
In Ω_1

$$K_\alpha(A \cup B)$$
$$= \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2})^\alpha e^{1-(\frac{1-\mu_B(x_i)+\nu_B(x_i)}{2})^\alpha}$$
$$+(1-\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2})^\alpha}-1]$$

For $0 < \alpha \leq 2$. Again from equation (2), we have

$$K_\alpha(A \cap B)$$
$$= \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[(\frac{\mu_{A\cap B}(x_i)+1-\nu_{A\cap B}(x_i)}{2})^\alpha e^{1-(\frac{1-\mu_{A\cap B}(x_i)+\nu_{A\cap B}(x_i)}{2})^\alpha}$$
$$+(1-\frac{\mu_{A\cap B}(x_i)+1-\nu_{A\cap B}(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_{A\cap B}(x_i)+1-\nu_{A\cap B}(x_i)}{2})^\alpha}-1]$$

For $0 < \alpha \leq 2$
In Ω_1

$$K_\alpha(A \cap B)$$
$$= \frac{1}{n(2^{1-\alpha}e^{1-2^{-\alpha}}-1)}\sum_{i=1}^{n}[(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(\frac{1-\mu_A(x_i)+\nu_A(x_i)}{2})^\alpha}$$
$$+(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha e^{1-(1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2})^\alpha}-1]$$

For $0 < \alpha \leq 2$. From equation (10) and (11), we get
$K_\alpha(A \cup B) + K_\alpha(A \cap B) = K_\alpha(A) + K_\alpha(B)$ in Ω_1. . Similarly the result holds in Ω_2. □

In the next section, we have checked the performance of proposed intuitionistic entropy measure on the basis of language variable given by De et al (2000).

5. Numerical Example

Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle, x \in \Omega\}$ be IFS for any $i \in R$ then
$A^n = \{\langle x_i, [\mu_A(x_i)]^n, 1-[1-\nu_A(x_i)]^n\rangle \mid x_i \in \Omega\}$ defined by De et al. (2000)

for IFS.
According to De et al. (2000) the classification of linguistic variables is given as, if A is considered as "LARGE" in Ω then $A^{\frac{1}{2}}, A^2, A^3$ and A^4 may be treated as "More or less LARGE", "Very LARGE", "Quite very LARGE" and "Very very LARGE" respectively. We use this to check performance of the proposed measure of intuitionistic fuzzy entropy. Mathematically a good intuitionistic fuzzy entropy measure should follow the given requirement by Li, Lu & Cai (2003):
$E(A^{\frac{1}{2}}) > E(A) > E(A^2) > E(A)^3 > E(A^4)$
Consider the IFS
$A = \{\langle 6, 0.1, 0.8\rangle, \langle 7, 0.3, 0.5\rangle, \langle 8, 0.5, 0.4\rangle, \langle 9, 0.9, 0\rangle, \langle 10, 1, 0\rangle\}$. For $0 \leq \alpha \leq 0.5$ the proposed intuitionistic fuzzy entropy measure satisfies $K_\alpha(A) > K_\alpha(A^{\frac{1}{2}}) > K_\alpha(A^2) > K_\alpha(A)^3) > K_\alpha(A^4)$. But for $0.5 < \alpha \leq 2$ the proposed intuitionistic fuzzy entropy measure satisfies $K_\alpha(A^{\frac{1}{2}}) > K_\alpha(A) > K_\alpha(A^2) > K_\alpha(A)^3) > K_\alpha(A^4)$ as given in table 1.

Table 1: Comparison of proposed entropy at different values of α

$K_\alpha(A^i)$	$K_\alpha(A^{\frac{1}{2}})$	$K_\alpha(A)$	$K_\alpha(A^2)$	$K_\alpha(A^3)$	$K_\alpha(A^4)$
$\alpha = 0.1$	0.788021	0.790307	0.782416	0.727757	0.749482
$\alpha = 0.5$	0.658281	0.664152	0.573514	0.48398	0.414756
$\alpha = 0.6$	0.630348	0.631765	0.523605	0.428614	0.360313
$\alpha = 0.9$	0.568236	0.553926	0.406565	0.304121	0.244213
$\alpha = 1.5$	0.499986	0.469539	0.279405	0.171569	0.13089
$\alpha = 1.7$	0.485269	0.397586	0.254913	0.146648	0.110962
$\alpha = 2.0$	0.46013	0.429463	0.216372	0.108377	0.081194

According to table 1, we can conclude that proposed measure of intuitionistic fuzzy entropy is consistent with the structured linguistic variables if value of α lies between 0.5 and 2.

6. Conclusion

In the present communication we have anticipated a α exponential intuitionistic fuzzy entropy for intuitionistic fuzzy sets corresponding to the entropy measure for fuzzy sets given by Gupta et al. (2014) with some properties. Further, the performance of proposed entropy measure at different values of α is compared with the help of an example. Although the maximum and minimum value of entropy is independent of values of α but the entropy measure is consistent with the structured linguistic variables if value of α lies between 0.5 and

2.

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